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Double quantum dot as a probe of nonequilibrium charge fluctuations at the quantum point contact

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Absorption of energy quanta generated by a quantum point contact results in the inelastic current through a double quantum dot placed nearby. In contrast to a single quantum dot, the inelastic current through the double quantum dot is determined by the nonlocal current correlations in the quantum point contact, which results in its sensitivity to the energy dependence of the quantum point contact transmission and can lead to suppression of the inelastic current for a substantial range of transport voltages on the quantum point contact. We calculate the inelastic current as a function of microscopic parameters of the circuit.

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An electronic system under nonequilibrium conditions generates fluctuations of electromagnetic field due to relaxation processes. The spectrum of those fluctuations carries valuable information about the microscopic properties of the system and its electronic distribution. Finding a proper way to measure the nonequilibrium noise becomes therefore an important subject of experimental and theoretical investigations.[1–3] Recently, a double quantum dot (DQD) has been proposed as a detector of nonequilibrium noise generated by nearby mesoscopic devices.[3] The noise detection is based on the generation of inelastic current through DQD assisted by absorption of energy quanta emitted by the device. To implement the noise measurement, DQD is brought into the Coulomb blockade regime with the highest-energy electron localized in one of its dots hereafter referred to as dot 1. Absorbing an energy quantum, an electron tunnels from the low-energy state in quantum dot 1 to the excited state localized in quantum dot 2, with the excitation energy Δ being fixed by the gate voltages. The tunnel barrier between the quantum dots is tuned to be much higher than the barriers between the dots and the adjacent leads so that after each interdot tunneling event the electron almost immediately escapes into the adjacent electron reservoir. Another electron occupies quantum dot 1, and the system returns to the ground state, with the unit of charge having been transferred through DQD.[1] The generated current is therefore proportional to the noise power on the excitation frequency $\Delta/\hbar$. Therefore, DQD realizes the frequency resolved noise detection and provides information about the nonequilibrium processes in the measured mesoscopic device. In view of this, it is important to have a precise relation between the inelastic current through DQD and the parameters of the mesoscopic circuit.

The idea of noise detection by DQD was experimentally realized in measurements of the nonequilibrium noise spectrum generated by a quantum point contact (QPC).[1,2] In those experiments, the QPC is brought in a strongly nonequilibrium regime by application of transport voltage. At the same time, the plunger voltage applied to the QPC controls its transmission. Theoretical calculations of the generated inelastic current have been performed in Ref. 3, where it has been related to the nonequilibrium noise power $S_I(\Delta/\hbar)$ generated by QPC at the frequency $\Delta/\hbar$. This noise power is given by the local current fluctuations in an arbitrary spatial point of QPC,

$$S_I^{\text{local}}(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle \delta I(x,\tau)\delta I(x,0)\rangle. \tag{1}$$

Based on the energy conservation law one concludes that when increasing the QPC transport voltage V_{QPC}, the current through DQD will start at the point $V^*_{\text{QPC}}=\Delta/e$, when the quanta with energy Δ appear in the nonequilibrium noise spectrum.[4] A puzzling feature of the experimental measurements is the independence of the threshold voltage V^*_{QPC} of the DQD excitation energy Δ for a finite range of energies, contrary to the expectations based on the energy conservation law.[1]

In this Rapid Communication we provide a theoretical description of QPC-DQD system in the nonequilibrium regime that allows to relate experimental measurements of the inelastic current to microscopic parameters. We show that since DQD is an object extended over both sides of QPC, local noise power (1) *is not the relevant quantity* for the inelastic DQD current. Rather, the noise power absorbed by DQD includes spatially nonlocal correlations of current fluctuations at positions of two quantum dots. The relevant voltage power is given by

$$\mathcal{S}_V(\omega) = \langle |Z(x_1,\omega)\delta\hat{I}(x_1,\omega) + Z(x_2,\omega)\delta\hat{I}(x_2,\omega)|^2\rangle, \tag{2}$$

where $Z(x_i,\omega)$ is the spatially dependent transimpedance of the circuit, relating the current fluctuations in the QPC part of the circuit to the fluctuations of electric potential at quantum dot i, x_i denotes the position of the quantum dot, and $\omega=\Delta/\hbar$ is the absorption frequency. The general expression of the inelastic current through DQD can be written as

$$I_{\text{DQD}} = \frac{e^3 t_0^2}{\hbar^2\Delta^2}\mathcal{S}_V\left(\frac{\Delta}{\hbar}\right), \tag{3}$$

where t_0 denotes the interdot tunneling amplitude. The voltage noise power S_V can be related to the direct current through QPC by an analogy of the Fano factor $S_V = 2eR_K^2\mathcal{F}_V(\omega)I_{\text{QPC}}$, where $R_K=h/e^2\approx 25,8$ kΩ is the quantum resistance. The explicit expression for $\mathcal{F}_V$ reads as

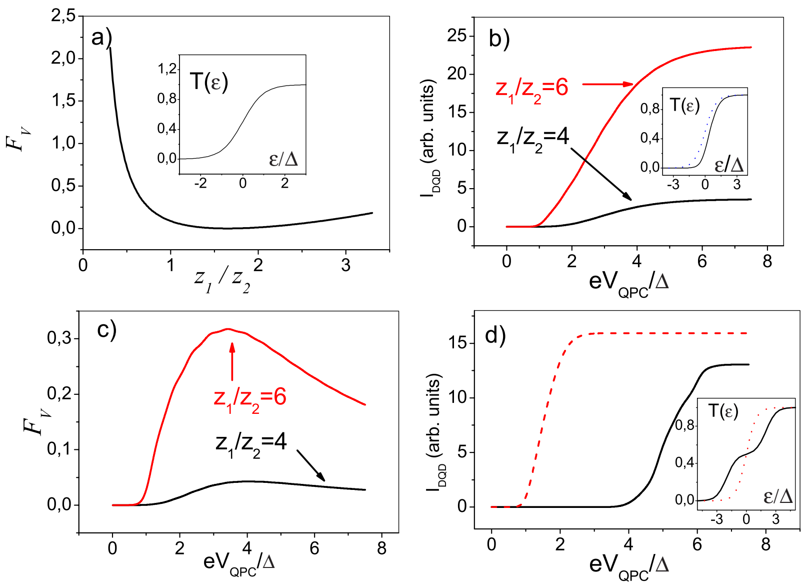

FIG. 1. (Color online) (a) Voltage Fano factor $F_V(\Delta/\hbar)$ as a function of transimpedance ratio z_1/z_2. Inset: energy dependence of QPC transmission $T(\epsilon)=|t(\epsilon)|^2$ resulting from Eq. (8). Panels (b) and (c) show the inelastic DQD current (b) and the Voltage Fano factor $F_V(\Delta/\hbar)$ (c) as a function of QPC transport voltage for $z_1/z_2=4$ and 6. Inset to panel (b): adopted energy dependence of QPC transmission $T(\epsilon)$ (solid line) deviates from the QPC transmission given by Eq. (8) (dots). (d) Inelastic DQD current vs QPC transport voltage for two different energy dependencies of QPC transmission. Inset: adopted energy dependencies of QPC transmission amplitude. Other parameters are relevant for the experiment in Ref. 1.

$$\mathcal{F}_V(\omega)=\left[1+\tanh\left(\frac{\hbar\omega}{2T}\right)\coth\left(\frac{eV-\hbar\omega}{2T}\right)\right]\times\frac{\int d\epsilon|z_1r_{\epsilon+\hbar\omega}t_\epsilon-z_2t_{\epsilon+\hbar\omega}r_\epsilon|^2(f^L_{\epsilon+\hbar\omega}-f^R_\epsilon)}{2\int d\epsilon|t_\epsilon|^2(f^L_\epsilon-f^R_\epsilon)}, \quad (4)$$

where t_ϵ and r_ϵ are the transmission and reflection amplitudes of QPC at energy ϵ and $z_i=Z(x_i,\omega)/R_K$ is the dimensionless transimpedance. The Fermi distributions in the left (source)/right (drain) reservoirs of QPC are denoted as $f_\epsilon^{L/R}$. Their chemical potentials differ by the QPC transport voltage and can be written as $\mu_{L/R}=\pm eV_{\mathrm{QPC}}/2$, with the chemical potential in the unbiased QPC being taken as zero. For zero temperature and for the absorption frequency $\omega=\Delta/\hbar$, Eq. (4) simplifies to

$$\mathcal{F}_V\left(\frac{\Delta}{\hbar}\right)=\Theta(eV-\Delta)\frac{\int_{-eV/2}^{eV/2-\Delta}d\epsilon|z_1r_{\epsilon+\Delta}t_\epsilon-z_2t_{\epsilon+\Delta}r_\epsilon|^2}{2\int_{-eV/2}^{eV/2}d\epsilon|t_\epsilon|^2}. \quad (5)$$

Using Fano factor (4), the expression for the generated inelastic current can be cast in the form of

$$I_{\mathrm{DQD}}=2eI_{\mathrm{QPC}}R_K^2\mathcal{F}_V(\Delta/\hbar)\Gamma_{\mathrm{DQD}}/\Delta^2, \quad (6)$$

where $\Gamma_{\mathrm{DQD}}=\frac{e^3t_0^2}{\hbar^2}$ is the DQD rectification factor.

The presence of nonlocal correlations substantially modifies the resulting noise spectrum. So, while the energy dependence of the QPC transmission is not essential for local current fluctuation (1),[3–5] it becomes crucial for the spatially nonlocal fluctuations of currents and corresponding potentials [Eq. (2)]. In particular, the noise can be substantially suppressed if the condition

$$z_1r_{\epsilon+\Delta}t_\epsilon-z_2t_{\epsilon+\Delta}r_\epsilon=0 \quad (7)$$

is fulfilled for energies ϵ within the transport voltage window. Let us assume that the energy dependence of the QPC transmission in the form

$$|t(\epsilon)|^2=[\exp(-\epsilon/W)+1]^{-1}, \quad (8)$$

where the energy scale W is related to the curvature of the QPC potential barrier. Then, substituting Eq. (8) into Eq. (7) we obtain the ratio of transimpedances at which the inelastic current through DQD is completely suppressed, $z_1/z_2=\exp[\Delta/(2W)]$. For this ratio of impedances condition (7) is fulfilled for each energy ϵ. Then the DQD current remains suppressed as long as the deviation of the QPC transmission from the assumed dependence [Eq. (8)] remains small for energies ϵ within the transport voltage window. The voltage Fano factor as a function of transimpedance ratio z_1/z_2 is shown in Fig. 1(a) for the QPC transmission given by Eq. (8). One can see that the nonlocal voltage fluctuations are suppressed at $z_1/z_2\approx1.7$.

The deviations of the QPC transmission from form (8) can be expected for the electron energies far from the top of the potential barrier. Those deviations can also be induced by the distortion of QPC potential barrier due to the applied transport voltage. Therefore, Eq. (7) determines the regime, when the threshold voltage V^*_{QPC} is a nonuniversal quantity. It is determined not by the energy conservation condition but rather by the form of the QPC potential barrier that itself is a nonlinear function of the applied transport voltage.

To illustrate those findings, the inelastic current through DQD and the voltage Fano factor as a function of QPC transport voltage are shown in Figs. 1(b) and 1(c). For panels (b) and (c) we assumed the energy dependence of QPC transmission $|t(\epsilon)|^2=[\exp(-\epsilon/W)+1]^{-2}$, which is different from Eq. (8). This dependence is shown in the inset to panel (b) by the solid line. Despite the deviation of the transmission from Eq. (8) condition (7) is fulfilled in a wide range of the energies ϵ for $z_1/z_2=4$. That results in a suppression of I_{DQD} and of the Fano factor in a wide range of QPC transport voltages exceeding the value Δ/e following from the energy conservation law. The DQD current starts at the voltage when the deviation of the QPC transmission from Eq. (8), and hence the violation of condition (7) becomes substantial. In contrast, for the value of $z_1/z_2=6$ that does not respect Eq. (7), the DQD current starts at $V_{\text{QPC}}=\Delta/e$ as follows from the energy conservation condition.

The suppression of DQD inelastic current can also result from plateaulike features in the energy dependence of QPC transmission. This is illustrated in Fig. 1(d), where DQD current is calculated for symmetric couplings $z_1=z_2$ and for two different dependencies $t(\epsilon)$ shown in the inset. The plateau in the $t(\epsilon)$ results in the suppression of DQD current in a range of QPC voltages substantially exceeding the energy conservation threshold.

In what follows we introduce the theoretical model for the coupled QPC-DQD system and outline the derivation of the presented results. Since the maximal inelastic current is observed when a new conducting channel is opening in QPC, we concentrate on a single conducting channel of QPC. We distinguish two species of electrons in QPC, namely, those coming from the right and the left reservoirs, and we describe them by the fermion field operators $\hat{\psi}_{R/L}(x)$.[6] The electrons of each sort are in equilibrium with its own reservoir. Taking the position of the QPC potential barrier at $x=0$, we represent the field on each side of it as

$$\hat{\psi}(x) = \int \frac{dk}{2\pi}\{\hat{\psi}_L(k)[e^{i(p_F+k)x} + r_\epsilon e^{-i(p_F+k)x}] + \hat{\psi}_R(k) t_\epsilon^* e^{-i(p_F+k)x}\}, \quad \text{for} \quad x<0, \tag{9}$$

$$\hat{\psi}(x) = \int \frac{dk}{2\pi}\{\hat{\psi}_R(k)[e^{-i(p_F+k)x} + r_\epsilon^* e^{i(p_F+k)x}] + \hat{\psi}_L(k) t_\epsilon e^{i(p_F+k)x}\}, \quad \text{for} \quad x>0. \tag{10}$$

Here p_F denotes the Fermi wave vector at zero transport voltage.

We assume that the quantum dots are situated far away from the potential barrier of QPC, one on each side of it (see experimental setup of Ref. 1). The structure of the interaction between DQD and QPC channels plays a crucial role for the generation of inelastic current, determining the transimpedance $Z(x_i,\omega)$. Due to the presence of external electrodes, the effective interaction becomes screened and time retarded. Moreover, the presence of QPC violates the spatial homogeneity of the interaction. Therefore, we can write the interaction term in the action in the form

$$\mathcal{A}_{\text{int}} = -e^2 \sum_{i=1,2} \int dx \int dt dt' \, U_i(|x|,|x-x_i|,t-t')\hat{n}(x,t)\hat{n}_i(t'). \tag{11}$$

Here $\hat{n}_1$ and $\hat{n}_2$ are the particle number operators in each quantum dot, and $\hat{n}(x)$ is the operator of density fluctuations in the conducting channel at the point x. The forward and backward inelastic scattering amplitudes in the QPC conducting channel are given in terms of Fourier transforms $\int dx e^{-iq(x-x_i)} U_i(|x|,|x-x_i|)$ at wave vectors $|q| \ll p_F$ and $q = \pm 2p_F$. We assume that the interaction is strongly screened, and it takes place only in a small region of the size of the screening length around each quantum dot. Then the behavior of scattering amplitudes at small wave vectors is smooth, and we can approximate $U_i(|q| \ll p_F) \approx U_i(q=0) \equiv U_i^f$ for the forward scattering. For the backward scattering we obtain $U_i^b(\pm 2p_F) = U_i(\pm 2p_F)e^{\mp 2ip_F x_i}$. Taking into account the finite size of a quantum dot, one has to integrate over x_i within that size, which greatly diminishes the backscattering amplitude because of the rapidly oscillating factors $e^{\pm 2ip_F x_i}$. On that account we neglect the backscattering amplitudes in what follows.

In the detection regime, the total occupation of DQD is fixed to $n_1(t)+n_2(t)=1$. This allows us to use a pseudospin 1/2 description of DQD. We associate the states localized in the quantum dots 1 and 2 with the spin-up and spin-down states, respectively. The charge transfer between the two quantum dots corresponds to the spin flip between the ground-state spin up and the excited state spin down. Interaction term (11) can be separated into the interaction with the total charge of DQD and the interaction with the z component of DQD pseudospin, $\hat{S}^z = \hat{n}_1 - \hat{n}_2$. Only the latter is relevant for the generation of DQD inelastic current. Note that the product $\hat{\pi} = e\hat{S}^z$ is proportional to the operator of the DQD dipole moment. Omitting the interaction with the total charge of DQD, we remain with the interaction between the dipole moment of DQD and dipole fluctuations of electric potential generated by QPC, which in the Fourier-transformed form reads as

$$\mathcal{A}_{\text{int}} = -\int \frac{d\omega}{2\pi} \hat{P}(\omega)\hat{\pi}(-\omega). \tag{12}$$

Here the dipole fluctuations of QPC potential $\hat{P}(\omega)$ are given by

$$\hat{P}(\omega) = e[U_1^f(\omega)\hat{n}(x_1,\omega) - U_2^f(\omega)\hat{n}(x_2,\omega)]. \tag{13}$$

Furthermore, introducing the spatially dependent transimpedance $Z(x_i,\omega) = \frac{1}{v_F}U_i^f$ and using the representation of the electric current operator in the basis [Eqs. (9) and (10)], we can express the dipole fluctuations $\hat{P}(\omega)$ in terms of the Fourier transform of the current operator at frequency ω,[7–9]

$$\hat{P}(\omega) = \sum_{i=1,2} Z(x_i,\omega)\hat{I}(x_i,\omega). \tag{14}$$

At this point it becomes evident that the dipole moment interacting with DQD involves spatially nonlocal correlations of QPC current. In the case of symmetric circuit, $U_1^f = U_2^f$, the transimpedance becomes independent of coordinate. Its expression in terms of the elements of effective electric circuit is provided in Ref. 3.

The generated inelastic current is calculated perturbatively in the lowest order of QPC-DQD interaction employing the Keldysh technique,[10,11] resulting in the final expression for DQD current (6). The diagrammatic representation of the first nonvanishing contribution to the inelastic current is shown in Fig. 2; the details of calculations are reported as a supplementary material.[9]

In conclusion we note that the generation of inelastic DQD current can be considered as a kind of a Coulomb drag experiment. Indeed, the diagrammatic representation for the inelastic current [Fig. 2(d)] looks very similar to the dia-

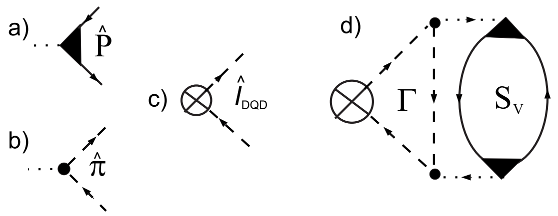

FIG. 2. Diagrammatic representation: (a) vertex of QPC dipole moment $\hat{P}$; (b) vertex of DQD dipole fluctuations $\hat{\pi}$; (c) DQD current vertex $\hat{I}_{\mathrm{DQD}}$; and (d) diagram for DQD current in the second order of interactions. S_V is nonlocal voltage noise (2), and Γ is the DQD rectification factor. Solid and dashed lines denote the one particle Green's functions in QPC and in DQD, respectively.

grams for the drag current.[8,12] To realize the drag, the particle-hole (p-h) symmetry has to be violated in both the drive (QPC) and the drain (DQD) parts of the circuit. The violation of the p-h symmetry in DQD is obvious because of the asymmetric charge distribution between the quantum dots. The violation of p-h symmetry in the QPC part of the circuit requires a more careful consideration because this part of the circuit is in strong nonequilibrium. This difference between the two parts of the circuit can be noticed in the form of the diagram in Fig. 2(d) for the generated inelastic current. The QPC part of the diagram consists of the nonequilibrium voltage noise operator S_V instead of the correlator of three current operators known from the perturbative calculations of the Coulomb drag.[12] Thus the asymmetry to the p-h transformation in the driving part of the circuit should be exhibited not by the fluctuations of the current through QPC but rather by the induced fluctuations of the voltage on DQD. This fact is reflected by the appearance of the transimpedances in Eq. (7). Moreover, since DQD-QPC circuit is a spatially inhomogeneous system, the interactions that transfer the energy of charge fluctuations between the drive and the drag parts of the circuit can depend on the position in space. This dependence is reflected in the different transimpedances z_1 and z_2. In that regard, relation (7) is understood as a condition of the particle-hole symmetry of the induced voltage fluctuations with frequency $\Delta/\hbar$, under which the Coulomb drag is suppressed. For the case of equal transimpedances condition (7) reduces to the condition of the particle-hole symmetry for the fluctuations of the QPC current.

To summarize, a crucial feature of DQD as a noise detector is its sensitivity to the nonlocal current fluctuations at QPC. The nonlocal spatial structure of the absorbed noise quanta can lead to the suppression of DQD current beyond the energy conservation threshold Δ/e as observed in experiment.[1] This effect represents a profound feature of the current voltage characteristics of QPC-DQD circuit that has no analogy in the shot-noise-induced current through a single quantum dot.[13] It may result from a special relation of local transimpedances (7) as well as from special features in the energy dependence of the QPC transmission. Condition (7) suggests the use of DQD as a measurement device for the energy dependence of QPC transmission.

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